

Schrodinger's Time Independent Equation.

we know that.

$$\text{Total Energy} = K.E + P.E \quad \left\{ K.E = \frac{1}{2} m v^2 = \frac{1}{2} \frac{m^2 v^2}{m} = \frac{p^2}{2m} \right\}$$

$$E = \frac{p^2}{2m} + V(x)$$

Multiply by ' $\psi(x)$ ' above equation,

$$E \psi(x) = \frac{p^2}{2m} \cdot \psi(x) + V(x) \cdot \psi(x) \quad \text{--- (1)}$$

General funⁿ. of $\psi(x)$ is

$$\psi(x) = e^{i(kx - \omega t)} \quad \text{--- (2)}$$

Differentiating eqⁿ (2) Twice w.r.t ' x '

$$\therefore \frac{d}{dx} \psi(x) = i \cdot k \cdot e^{i(kx - \omega t)}$$

$$\therefore \frac{d^2}{dx^2} \psi(x) = i^2 \cdot k^2 \cdot e^{i(kx - \omega t)} = -k^2 \cdot \psi(x)$$

$$\text{Here, } k = \frac{p}{\hbar}$$

$$\therefore \frac{d^2}{dx^2} \psi(x) = - \frac{p^2}{\hbar^2} \psi(x) = - \frac{p^2}{\hbar^2} \psi(x)$$

$$\therefore p^2 \psi(x) = -\hbar^2 \frac{d^2}{dx^2} \psi(x) \quad \text{--- (3)}$$

Put eqⁿ (3) in eqⁿ (1)

$$E \psi(x) = - \frac{\hbar^2}{2m} \cdot \frac{d^2}{dx^2} \psi(x) + V(x) \cdot \psi(x)$$

$$\therefore E \cdot \psi(x) - V(x) \cdot \psi(x) + \frac{\hbar^2}{2m} \cdot \frac{d^2}{dx^2} \psi(x) = 0$$

Multiply by ' $2m$ ' and divide by ' $\hbar^2$ ' above equation,

$$\frac{2m}{\hbar^2} E \cdot \psi(x) - \frac{2m}{\hbar^2} V(x) \cdot \psi(x) + \frac{d^2}{dx^2} \psi(x) = 0$$

$$\therefore \frac{d^2}{dx^2} \psi(x) + \frac{2m}{\hbar^2} [E \psi(x) - V(x) \cdot \psi(x)] = 0$$

This equation is one dimensional
 ——— x ———